RUSSIAN RESEARCH CENTRE KURCHATOV INSTITUTE
РОССИЙСКИЙ НАУЧНЫЙ ЦЕНТР "КУРЧАТОВСКИЙ ИНСТИТУТ"

Extension of conventional MHD equilibrium theory to model the fast particle effects

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5th ITER International Summer School
Aix en Provence, France, June 20 - 24, 2011

Motion of a single particle

$$m_p \frac{d\mathbf{v}_p}{dt} = \mathbf{F} = q_p(\mathbf{E} + \mathbf{v}_p \times \mathbf{B})$$

depends on the electric and magnetic fields E and B created by all other particles and external sources

$$= \frac{\quad}{\quad} \quad ' \times \frac{- \;'}{\left| - \;' \right|} dV'$$

Standadrd MHD equations

Force balance: $\rho d\mathbf{v}/dt = -\nabla p + \mathbf{j}\times\mathbf{B}$ **with**

$\dfrac{d}{dt} = \dfrac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$ **in equilibrium** $\nabla p = \mathbf{j}\times\mathbf{B}$

Maxwell eqns: $\nabla\cdot\mathbf{B} = 0$, $\nabla\times\mathbf{B} = \mu_0\mathbf{j}$, $\nabla\times\mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial t}$

& sometimes $\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$ **magnetic flux conservation**

Currents in the equilibrium plasma

With $\nabla p = \mathbf{j}\times\mathbf{B}$ in equilibrium

we have, $\mathbf{j}_\perp = \dfrac{\mathbf{B}\times\nabla p}{\mathbf{B}^2}$, $\mathbf{j} = \mathbf{j}_\perp + \mathbf{j}_\parallel$ and

$\nabla\cdot\mathbf{j} = 0$

$$\cdot\cdot\mathbf{j}_\parallel = -\nabla\cdot\mathbf{j} \quad = \frac{\mathbf{B}\times\nabla}{\mathbf{B}^4}\cdot\nabla\mathbf{B}^2$$

Alternative: kinetic approach

Boltzmann eq:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{\mathbf{F}}{m_p} \cdot \nabla_{\mathbf{v}} f = \frac{\delta f}{\delta t}_{coll}$$

when averaged:

Distribution of **fast ions** produced by additional heating systems

"is **strongly anisotropic**,

with the NBI produced **fast ions** flowing predominantly **parallel** to the magnetic field, and the ICRH **accelerated ions** characterized by large **perpendicular** energy and mostly trapped orbits"

Fasoli A., et al., Nucl. Fusion **47** S264 (2007).

'Progress in the ITER Physics Basis'

Chapter 5: Physics of energetic ions

With such fast ions $\overleftrightarrow{p} \neq p\overleftrightarrow{\mathbf{I}}$ **and** $\nabla \cdot \overleftrightarrow{p} \neq \nabla p$

Then we assume

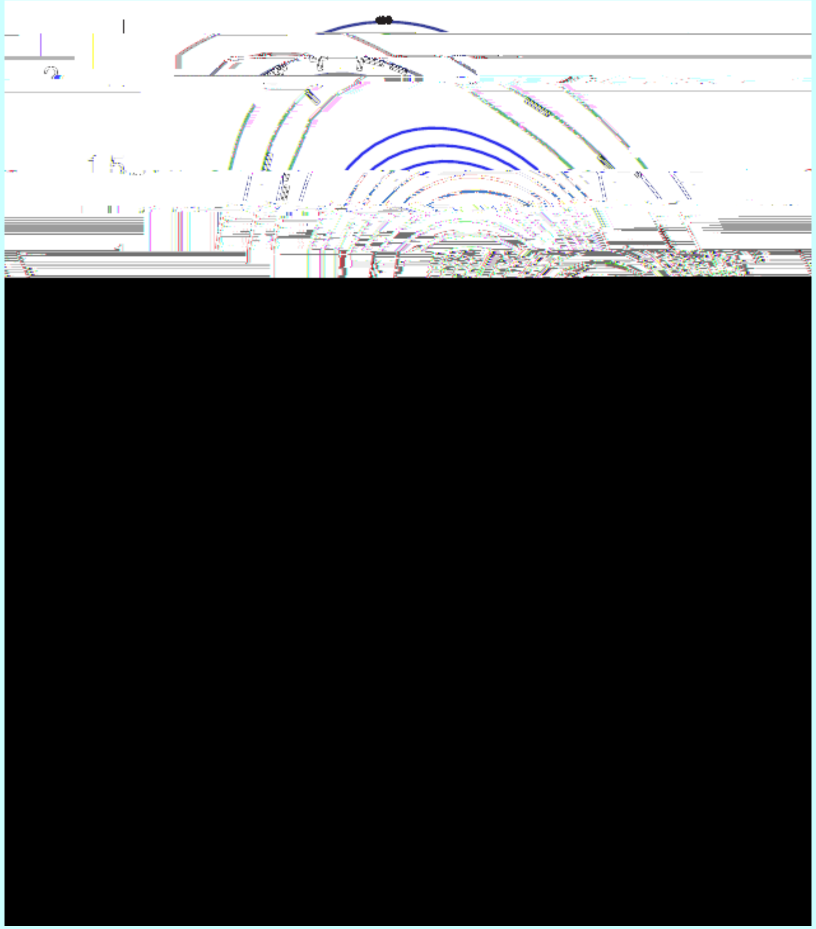
$$\overleftrightarrow{p} = p_{\parallel} \frac{\mathbf{BB}}{\mathbf{B}^2} + p_{\perp} \left(\overleftrightarrow{\mathbf{I}} - \frac{\mathbf{BB}}{\mathbf{B}^2} \right),$$

the most simple form of the pressure tensor with anisotropy.

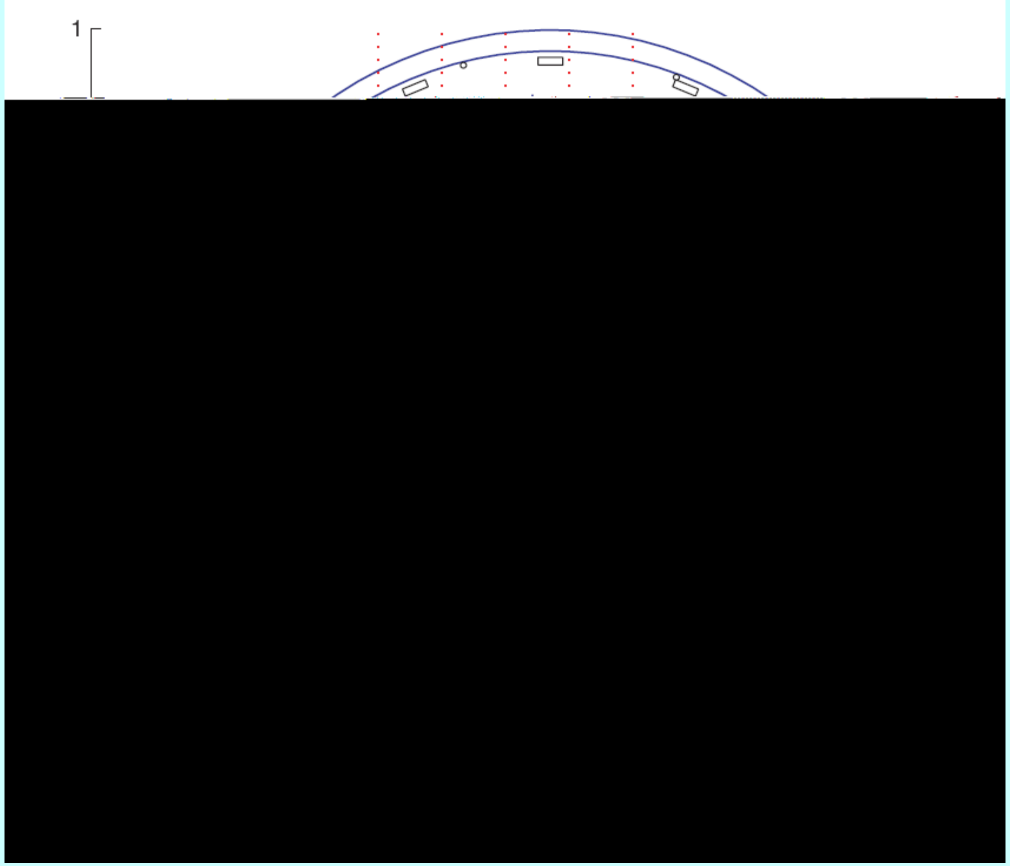
$$(p_{\parallel}, p_{\perp}) = \int m_p \left(v_{\parallel}^2, \frac{v_{\perp}^2}{2}\right) f d\mathbf{v}_p ,$$

parallel and perpendicular pressures

Examples from Zwingmann et al 2001 *PPCF* **43** 1441



JET: $p_{\|}$ **and** $a = \text{const}$



Tore Supra: $p_{\perp}$ **and** $a = \text{const}$

General relations

Start from general equilibrium equations

$$\nabla \cdot \overleftrightarrow{p} = \mathbf{j} \times \mathbf{B}, \quad \mu_0 \mathbf{j} = \nabla \times \mathbf{B}$$

Most popular assumptions

$$p_{\|} = p_{\|}(a,B), \qquad p_{\perp} = p_{\perp}(a,B)$$

with $a = \text{const}$ **the flux coordinate:** $\mathbf{B}\cdot\nabla a = 0$

1. **Good for symmetry (tokamaks),**
2. **Corresponds to the leading order solution of the Fokker–Planck equation for the distribution function** f **(which is** $\mathbf{B}\cdot\nabla f = 0$ **in this case)**

Other models? Better choice of $p_{\|}$ **and** $p_{\perp}$**?**

Examples of $p_{\parallel}$ and $p_{\perp}$ prescription

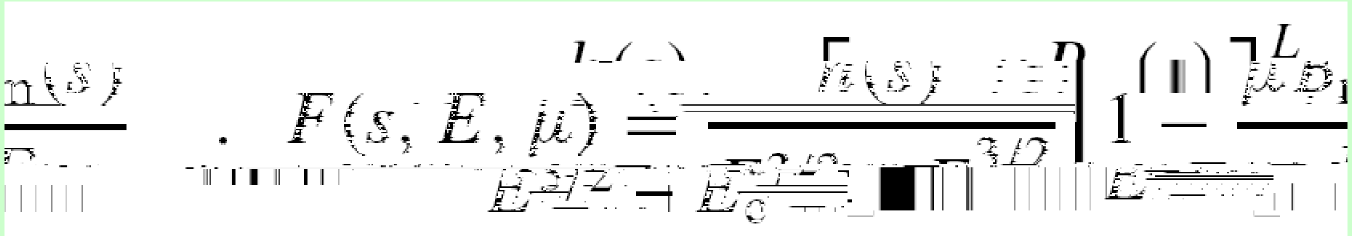
Zwingmann W, Eriksson L G and Stubberfield P, **Equilibrium analysis of tokamak discharges with anisotropic pressure**, 2001 *Plasma Phys. Control. Fusion* **43** 1441

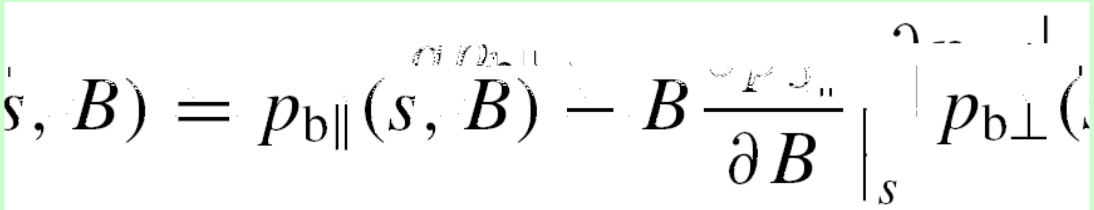
$$[illegible] \sum_{k=1}^{NP} [illegible] \sum_{k=1}^{NP} \sum_{n=1}^{NA} [illegible]$$

$$P_{\perp}(\Psi, R) = P_{\parallel}(\Psi, R) + R\frac{\partial P_{\parallel}(\Psi, R)}{\partial R}$$

Examples of $p_{\parallel}$ and $p_{\perp}$ prescription

Cooper W A *et al* 2005, **Three-dimensional** **anisotropic** **pressure equilibria that model balanced tangential neutral beam injection effects,** *Plasma Phys. Control. Fusion* **47** 561



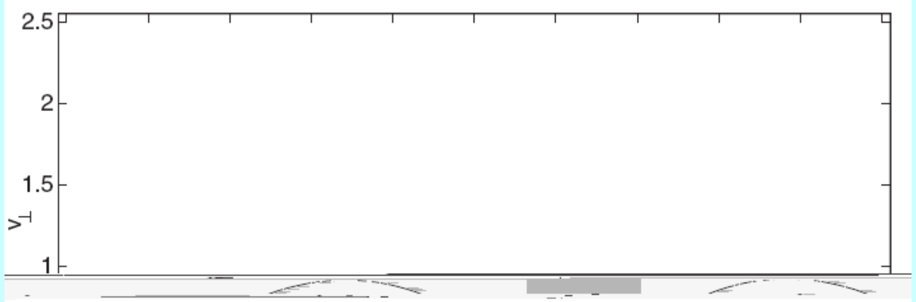


"modified slowing down distribution"

"model the effects of balanced tangential neutral beam injection"

Examples: Contours of constant f

model for balanced tangential NBI



Cr0Bper

Parallel force balance

$$\nabla p_{\|} = \sigma_{\|} \nabla (\mathbf{B}^2/2) + \mathbf{K} \times \mathbf{B}$$

$$\mathbf{B} \cdot \nabla p_{\|} = \sigma_{\|} \mathbf{B} \cdot \nabla (\mathbf{B}^2/2)$$

which is equivalent to

$$\mathbf{B} \cdot \nabla (p_{\|} + p_{\perp}) = -\mathbf{B}^2 \mathbf{B} \cdot \nabla \sigma_{\|}$$

We have

Parallel force balance: consequences

$$p_{\|} \approx p_{\|0} + \frac{p_{\|0} - p_{\perp}}{2} \; 1 - \frac{\mathbf{B}_0^2}{\mathbf{B}^2}$$

with $p_{\|0} = p_{\|0}(a)$

in tokamaks and stellarators, $p_{\|} - p_{\|0}(a)$

must be small even at large variations of $p_{\perp}$**.**

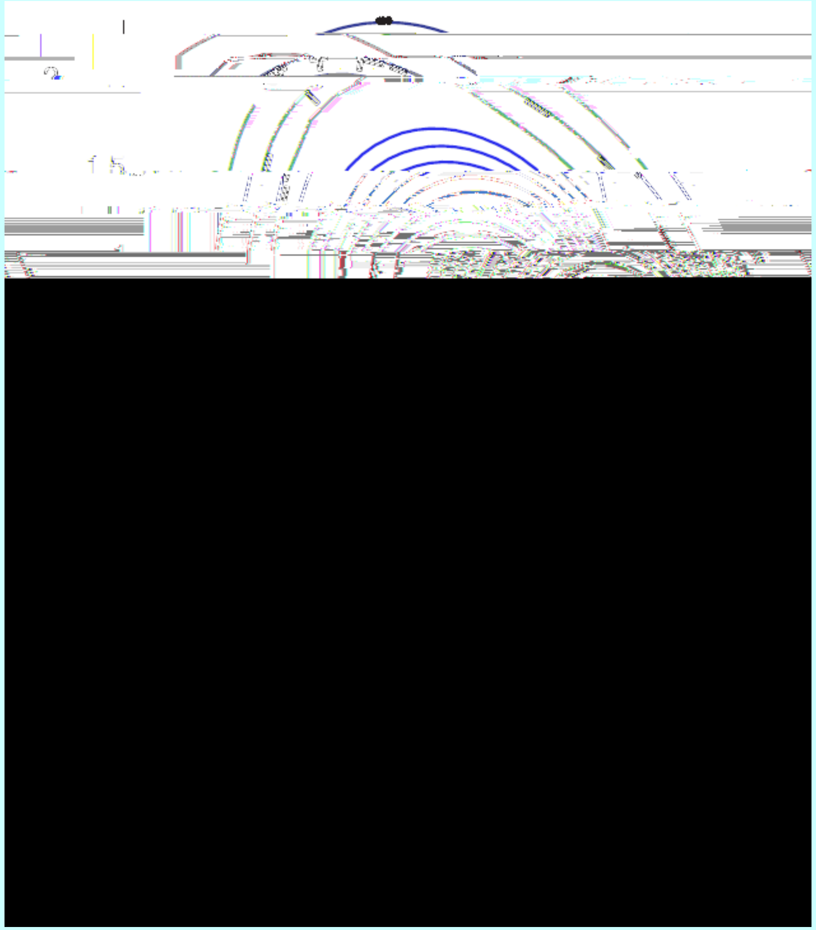
$$p_{\|} = p_{\|0} + \tilde{p}_{\|}$$

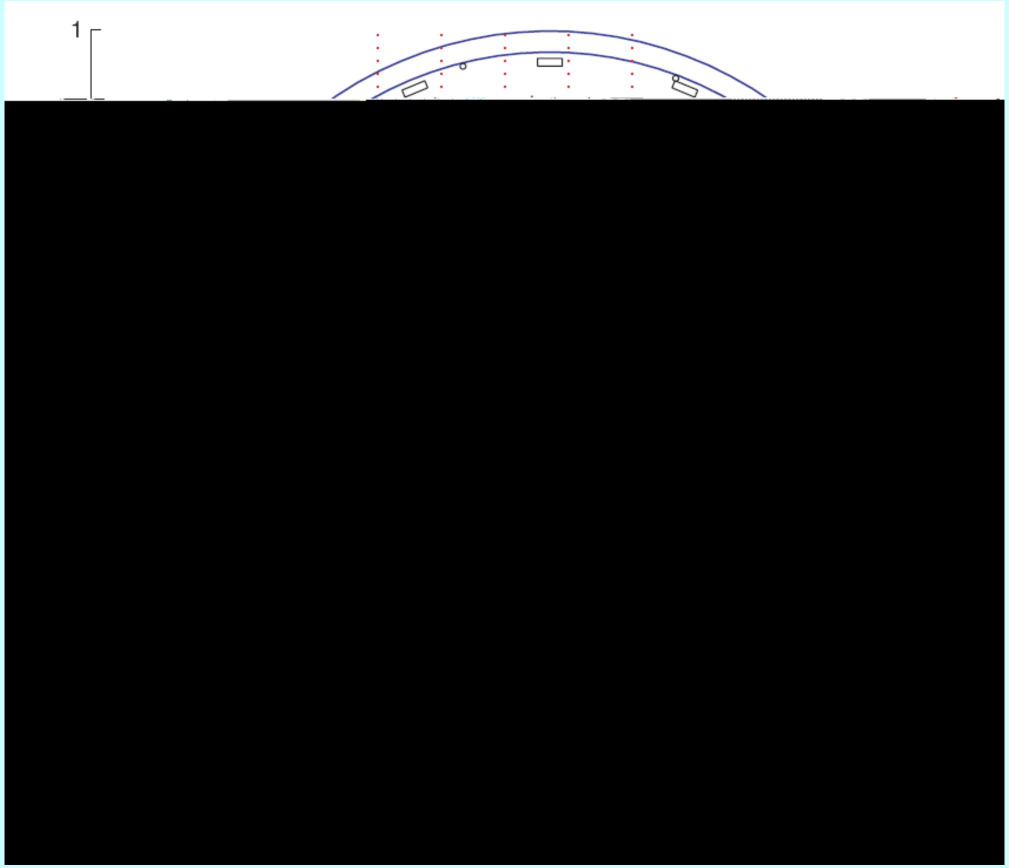
Large $\tilde{p}_{\|}$ **can be produced by very large** $\tilde{p}_{\perp}$ ***only*.**

Coop

“the

Examples from Zwingmann et al 2001 *PPCF* **43** 1441





JET: $p_{||}$ **and** $a = \text{const}$

Tore Supra: $\perp$

Perpendicular force balance

Poloidal ψ and toroidal Φ magnetic fluxes associated with a toroidal magnetic surface

$\equiv \qquad \cdot$

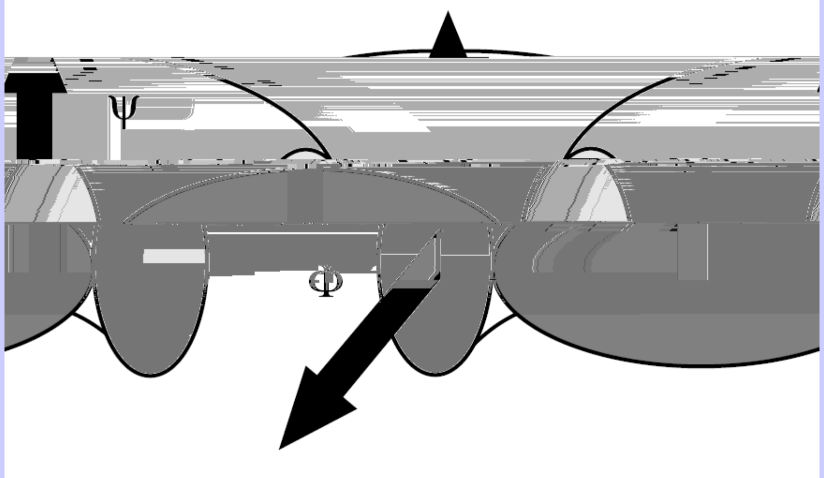
Magnetic diagnostics

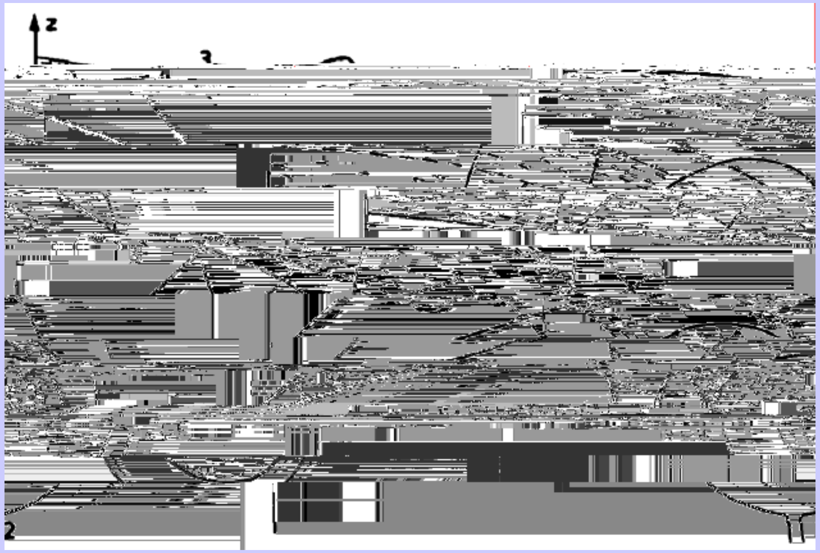
ψ is determined by $\mathbf{j}_{||}$,

by $p_{||} + p_{\perp}$

Φ is determined by $\mathbf{j}_{\perp}$,

by $p_{\perp}$





Experimental Results

Nucl. Fusion 45 (2005) L33–L36

Y. Narushima[1,2], K. Narihara[2], T. Tokuzawa[1,2], K. Tanaka[2],
ki[3] and LHD Experimental Group[2]

"In low density discharges of a Large Helical Device (LHD), **anisotropic pressure** is expected because the LHD has powerful tangential neutral beam injection systems.

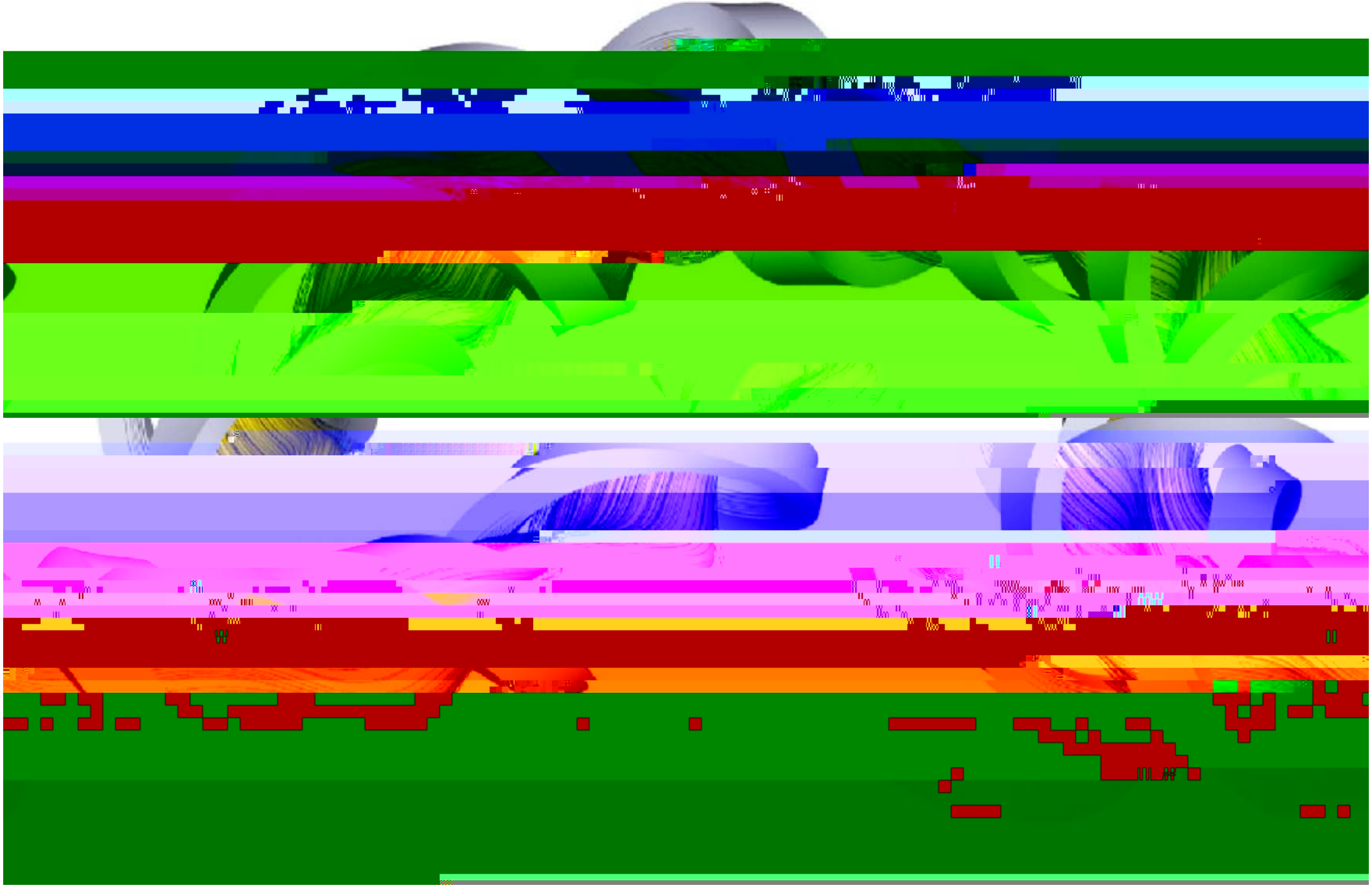
We show the strong correlation between the **pressure anisotropy** due to the beam pressure based on Monte Carlo calculations and the ratio of the diamagnetic loop signal and the saddle loop signal."

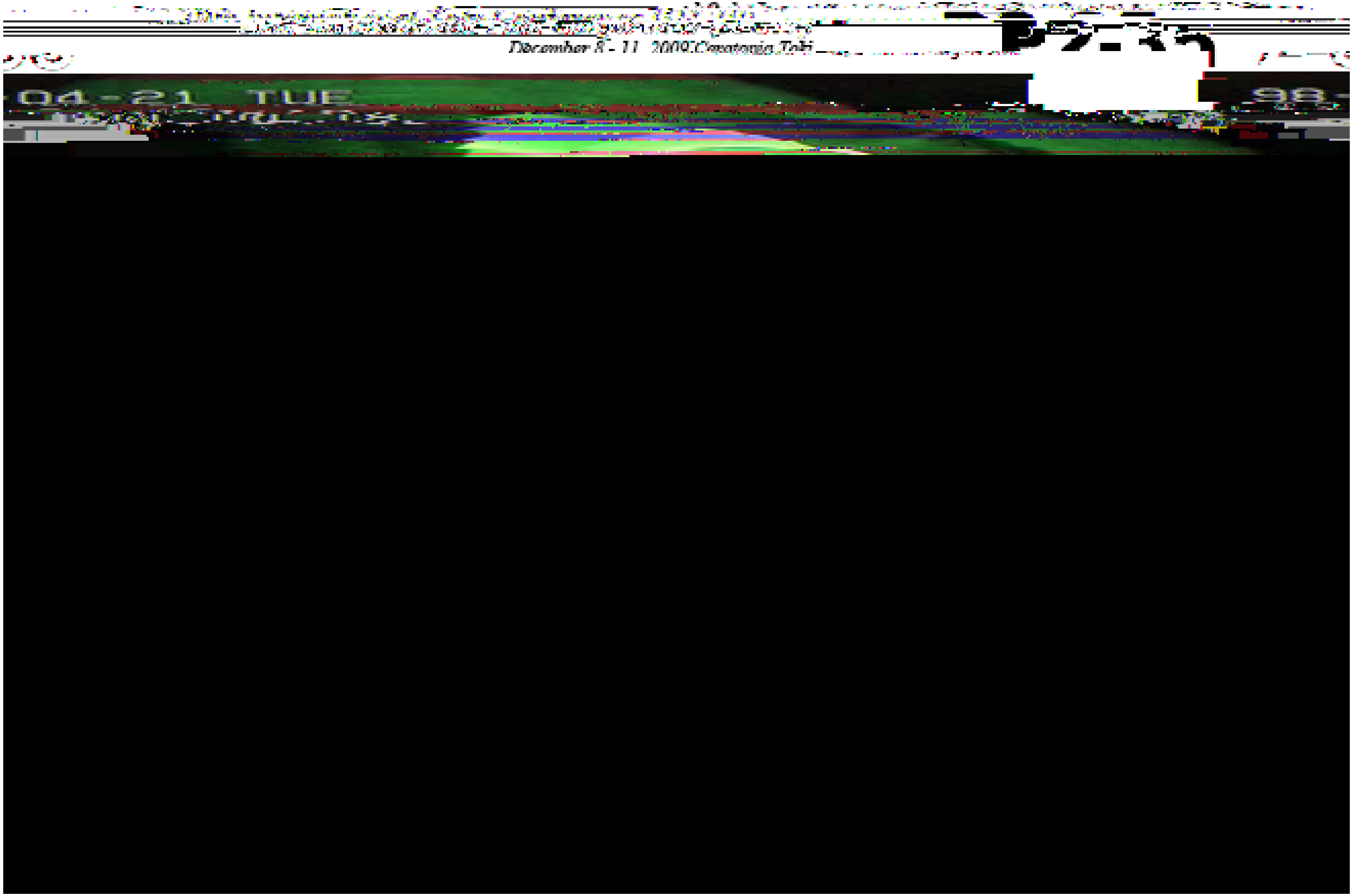
Large Helical Device (LHD)

All superconducting coil

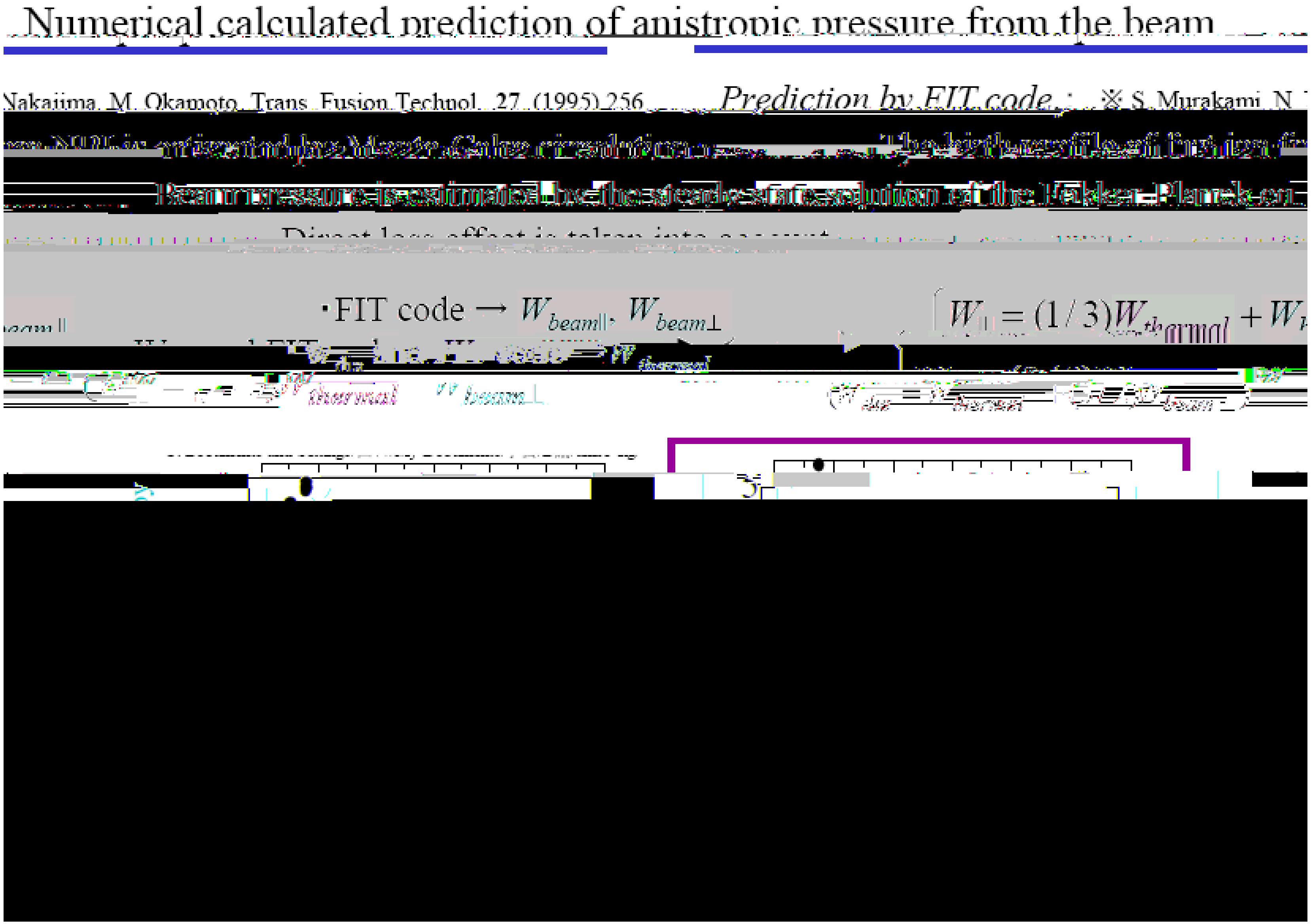
sysf1all su..i62T32td¶j0 3.228 TD.00

sysf1all su..i525m0d¶i0 3.228 TD0

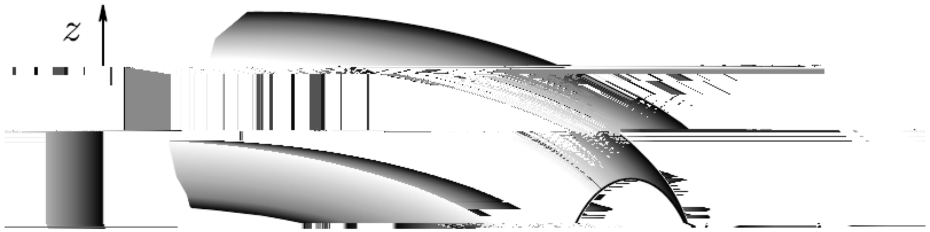




K.Y. Watanabe, et al., P2-35, ITC-2009



Plasma equilibrium with toroidal rotation



$$\rho(\mathbf{v}\nabla)\mathbf{v} = -\nabla\overleftrightarrow{p} + \mathbf{j}\times\mathbf{B}$$

Plasma with toroidal rotation, estimates

Proton mass

$$m_p = 1.67\times10^{-27}\ \ \text{kg}$$

Plasma density

$$n = 10^{20}\ \ \text{m}^{-3}$$

mass density

$$\rho = m_p n = 1.67\times10^{-7}\ \ \text{kg/m}^3$$

Compare to

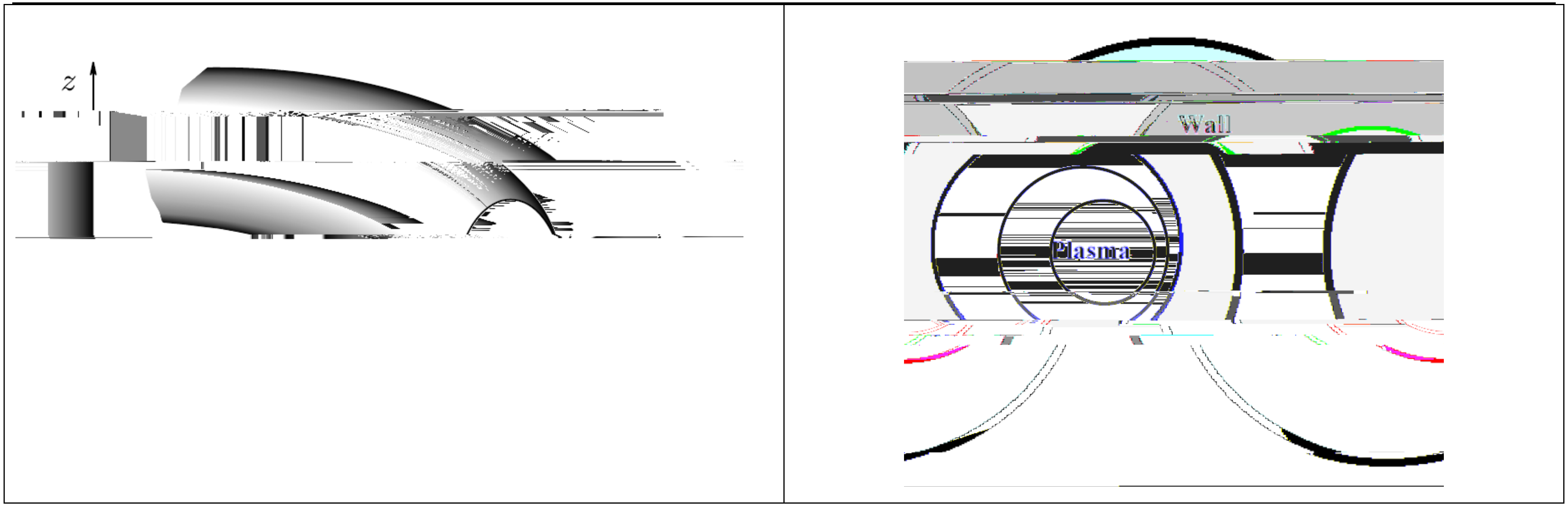
water $\rho = 10^3\ \ \text{kg/m}^3$

air $\rho = 1.29\ \ \text{kg/m}^3$

ITER: plasma volume $V_{plasma} = 870\ \ \text{m}^3$

Mass of H plasma $M = \rho V_{plasma} = 1.45\times10^{-4}\ \ \text{kg}$

Rotation and Shafranov shift



$$\Delta' = \Delta'_S - \frac{a}{R}\frac{\overline{\rho v_t^2} - \rho v_t^2}{B_\theta^2}$$ **with** $$\Delta'_S = -\frac{a}{R}\left(\frac{l_i}{2} + 2\frac{\overline{p} - p}{B_\theta^2}\right)$$

with $$l_i \equiv \overline{B_\theta^2} / B_\theta^2$$ **and** $$\overline{X} \equiv \frac{2}{a^2}\int_0^a X\rho d\rho$$

Rotation and Shafranov shift - 2

$$\Delta'(b) = -\frac{b}{R}\ \frac{l_i}{2} + \frac{2\bar{p} + \overline{\rho v_t^2}}{B_J^2}$$

The global effect of toroidal rotation is larger outward shift, but only weak increase

Effect comparable to pressure at $v_t \sim v_{T_i}$ **,**

or $v_{beam} \sim v_{T_i} \dfrac{S_{plasma}}{S_{beam}}$ **for a beam**

Summary

ü **Fast particles create the pressure anisotropy and rotation**

ü **In equilibrium, the deviations from conventional MHD must be mainly related to** $p_\perp$

$$p_\parallel\left(1+\frac{\mathbf{B}_0^2}{\mathbf{B}^2}\right)+p_\perp\left(1-\frac{\mathbf{B}_0^2}{\mathbf{B}^2}\right)\approx 2p_{\parallel 0}$$

ü **In some cases it must be possible to estimate the degree of pressure anisotropy by magnetic measurements**

$\mathbf{j}_\perp$ **is determined by** $p_\perp$ **, while** $\mathbf{j}_\parallel$ **is ~ determined by** $p_\parallel + p_\perp$

ü **Reliable when** $p_\perp << p_\parallel$ **or** $p_\perp \approx p_{\perp 0}(a)$

[redacted]

For more details see

1. Pustovitov V.D., **Equilibrium of Rotating and Nonrotating Plasmas in Tokamaks,** *Plasma Physics Reports* **29** (2003) p. 105.

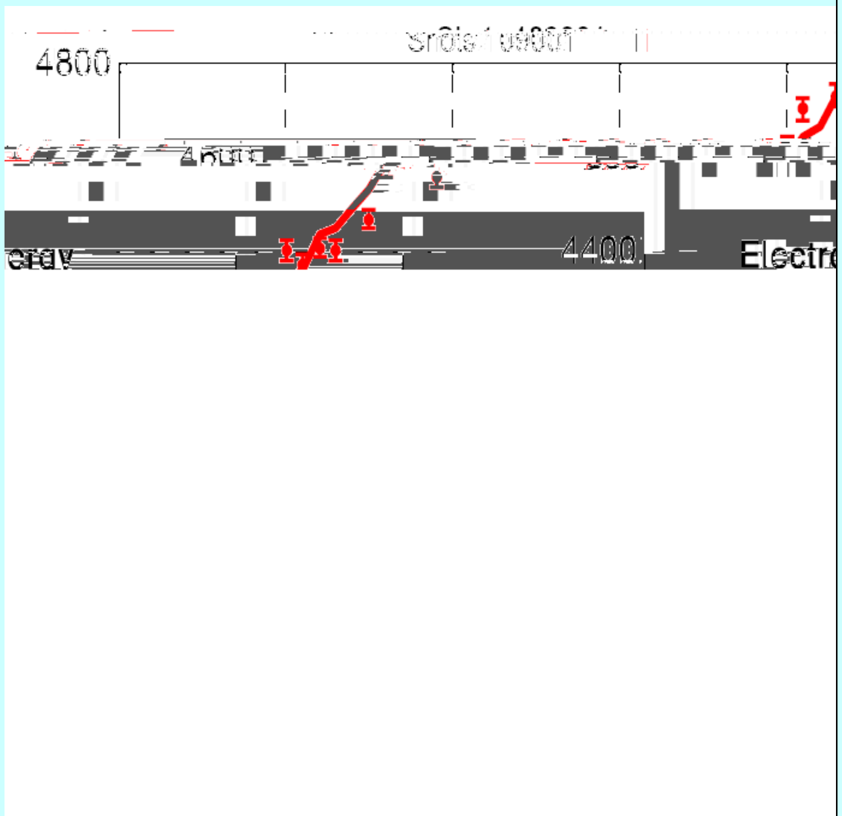
Backup slides

Experiments on T-10

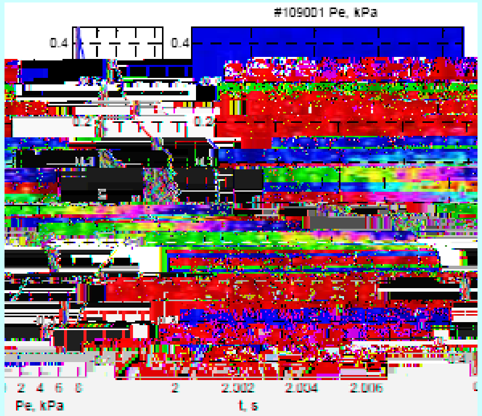
V.F. Andreev, et al., "*The ballistic jump of the total heat flux after ECRH switching on in the T-10 tokamak*" Plasma Phys. Control. Fusion **46**, 319 (2004).

Experiments on TEXTOR

M.Yu. Kantor, et al., "Thomson scattering diagnostic for study fast events in the TEXTOR plasma" *36th EPS Conf. Plasma Phys. (Sofia, June 29 – July 3 2009)* P-1.184



Electron heating inside q=1 surface during ECRH



"the absorbed energy is perfectly confined inside the q=1 surface during the first 5 ms"

"the electron heating rate inside the q=1 surface calculated from the local TS data shows ~200 kW which is

only one third

of the launched EC power"

Equations for the heat transport in

V.F. Andreev, et al., Plasma Phys. Control. Fusion **46**, 319 (2004).

Our main equations in

[3] V.D. Pustovitov and S.A. Stepanyan, Plasma Phys. Control. Fusion **53**, 035004 (2011). [4] V.D. Pustovitov, Plasma Phys. Rep. **37**, 109 (2011).

Force balance: $\rho d\mathbf{v}/dt = -\nabla p + \mathbf{j}\times\mathbf{B}$ $\nabla p = \mathbf{j}\times\mathbf{B}$

Integral energy balance

Integrate — — —— — *s*